Strategic Shirking in Promotion Tournaments

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Abstract

We provide a theoretical analysis of promotion tournaments in which workers "strategically shirk" by purposely under-performing on tasks that are de-emphasized in a promotion rule, while over-performing in tasks that are emphasized in the rule, thereby increasing their chances of promotion and a wage increase. To mitigate the multitasking problem, the firm might commit upfront to a promotion rule that requires more balance in the performances across tasks in a job than would otherwise be justified on productivity grounds. The analysis shows how "Putt’s Law" (which states that competent workers are sometimes passed over for promotion in favor of incompetent ones) can be understood as a natural consequence of the firm designing optimal promotion rules.

Keywords: tournaments, promotions, strategic shirking, Putt’s Law

JEL Classification: M53, J24

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1 Introduction

Starting with Lazear and Rosen (1981), a large literature on tournament theory has evolved concerning the role of promotions in creating incentives.\(^1\) The fundamental idea is that the prospect of getting promoted and receiving a higher wage motivates workers to exert more effort or to invest more heavily in acquiring human capital, because such actions are performance enhancing and therefore improve a worker’s promotion chances. In the standard theoretical models, shirking is not an attractive option for workers, because shirkers are likely to be either fired or outperformed by others who ultimately win a scarce number of promotions. But these results arise only because worker choices (and ultimately their performances) are assumed to be one dimensional, so that there is only one way to impress the boss. In reality, workers participating in promotion tournaments are engaged in multiple tasks, and weak performance on one task can be compensated by stellar performance on another. In such settings, a career-oriented worker may find it optimal to purposefully under-perform - i.e. "strategically shirk" - on one task, while over-performing on another, so as to influence future job assignments.

Our goal in this study is to explore theoretically the implications of strategic shirking in promotion tournaments. The main idea hinges on how promotions cause a particular type of misalignment between the interests of workers and the firm. Workers at low levels of the job ladder over-perform on tasks that are heavily valued at higher levels of the ladder (and therefore that the firm gives considerable weight to when making promotion decisions) and strategically shirk on other tasks that the firm would prefer they emphasize. A salesperson might, for example, focus more on demonstrating his leadership aptitude than on the main function of his current job, which is making sales. Although from a productivity standpoint the firm would like the workers at a lower rung of the job ladder to weight various tasks in a particular way, those workers will

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instead behave according to the weights dictated in the promotion rule, so as to strengthen their case for promotion. This multitasking problem, though fundamental, has been neglected in the literature on promotion tournaments.

Our model describes a setting in which employers design promotion systems with both incentives and job assignment in mind. There are two workers and one firm, all risk neutral. There are two periods and a two-level job hierarchy in which each of the two jobs has the same two tasks, though the tasks vary in their relative importance (i.e. contribution to output) across jobs. Output is determined by effort and ability, both of which are task-specific.\(^2\) Initially, the firm is ignorant of worker abilities when the workers begin their careers in the low-level job. After a period of work, the firm observes the relative performances of the workers on each task, and on the basis of those observations the firm promotes one of the workers to the high-level job and retains the other in the low-level job. The promotion rule is based on a weighted average of the relative performance signals on each task, where the firm commits to the weights upfront and communicates them to the workers. As usual, the workers dislike effort but are motivated by the wage increase attached to promotion, and the weights the firm assigns in the promotion rule shape the workers’ first-period allocation of effort across the two tasks. In particular the workers strategically shirk on the task that is under-emphasized in the promotion rule, even if it is the firm’s preferred task on productivity grounds. The difficulty the firm faces is that it would like workers in the low-level job to emphasize the task that is relatively more productive in that job, but from the standpoint of second-period productivity the firm wants to be sure that the worker who is promoted is the one who is relatively better at the task emphasized in the high-level job. The natural promotion rule would lead workers in the low-level job to overemphasize the task that is most highly valued in the high-level job and that is therefore weighted heavily in the promotion rule. To mitigate this incentive problem, the firm might choose the promotion rule in a way that puts less (more) weight on the task emphasized in the high-level (low-level) job, compared to what it would do in the absence of an incentive problem. The main result is greater balance between the tasks in the stated promotion rule than what would be expected strictly on productivity grounds and absent incentive considerations.

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\(^2\)Recent theoretical and empirical literature has recognized the importance of the concept of task-specific skills, both within occupations and in the context of job hierarchies. See, for example, Gibbons and Waldman (2004, 2006), Poletaev and Robinson (2008), Brilon (2010), Gathmann and Schönberg (2010), Cassidy (2012), and Yamaguchi (2012).
This also suggests that in the interests of preserving incentives the firm may wish to promote people who have invested in a broad range of skills, even those not heavily emphasized in the higher-level job.

Broadening jobs to include multiple productive tasks - as opposed to a single task as is standard in the tournament literature - allows us to explain an interesting (and at first glance puzzling) aspect of promotion decisions. Although we typically think that promotions are merit-based and that excellence in one's current position will ultimately be rewarded by future promotion, in practice this does not always happen. For a variety of reasons, such as organizational politics or outright mistakes by the employer, a worker who excels in a given job is sometimes passed over for promotion in favor of a colleague whose performance in that same job seems less impressive. In fact, as we document later, anecdotal evidence supports the view that in some cases promotions are denied precisely because someone excels in their job. That is, some workers may simply be too good to promote out of their current jobs. Ironically, by striving for excellence in the job at hand, a worker might condemn himself to remain in it forever by making himself "indispensable". While the concept has been neglected in the economics literature, it is sufficiently popular that it has acquired its own name in management circles - Putt's Law. The "Law" is typically defined by the following quote:

"Technology is dominated by two types of people: those who understand what they do not manage and those who manage what they do not understand." - Archibald Putt (2006)

According to Putt's Law, those who are promoted are often not those who have demonstrated the greatest talents. We show how Putt's Law and its implications can be understood in the context of an economic model in which firms optimally design promotion systems. Putt's Law is one observable implication of the model. Another is balance (across tasks) in the promotion rule, and others relating to balance are based on the degree to which jobs differ (in the productivity weights attached to tasks) across hierarchical levels. Yet another implication, which arises from an extension of our model, is that promotion tournaments and output-based pay contracts like bonuses or piece rates go hand in hand. We discuss all of these testable implications in Section 5.

Our analysis relates to DeVaro and Gürtler (2012), which introduces the concept of strategic shirking in a setting in which there is no job hierarchy and therefore no promotions. That analysis provides a theoretical explanation for the practice of multitasking workers even after the firm has observed their
task-specific productivities. The main result is that committing early on to a practice of late-stage multitasking can be optimal in that it mitigates the incentive problem caused by strategic shirking. The result in the present paper that (in response to the threat of strategic shirking) the firm may opt to commit to a promotion rule that involves more balance between the two tasks echoes the multitasking result from the earlier paper. In the earlier analysis workers "overwork" on a preferred task and strategically shirk on a less favored one. Similar behavior occurs in the present analysis, though the reason one task is preferred is not because of exogenous worker preferences over tasks but rather because showing promise on that task increases the likelihood of a wage-enhancing promotion, because it directly links to skills that are highly valued by the firm in the high-level job.

The analysis also relates to the large literature on promotion tournaments, most of which assumes a unidimensional productivity-enhancing worker choice variable (either a human capital investment or, more commonly, an effort choice). Whereas the literature's focus is on how the optimal effort level compares to the first best, ours is on the distortion of effort across multiple tasks in a given job. The branch of the tournament theory literature concerning sabotage is also related in that workers can allocate their attention to both productive work (i.e. effort) and unproductive or destructive work (i.e. sabotage).

Our analysis also relates to a small number of prior studies considering tournaments with multiple activities. In Clark and Konrad (2007), worker effort is chosen at constant marginal cost and spread over \( n \) different dimensions, and a given worker must win at least \( k \) of those dimensions to win the tournament prize. In their model, as in the standard tournament literature with unidimensional contests, the principal receives only information on the identity of the winner rather than information specifying the margin of victory on each of the \( n \) dimensions. In contrast, in Franckx, D'Amato, and Brose (2004) the employer receives a signal in each of the competitive dimensions, as in our model the employer observes a relative performance signal on each task in the lower-level job. In contrast to these models, in ours there is strategic shirking and also employer learning about worker task-specific abilities, and second-period job assignments are made on the basis of this information.

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4 See also the model of Prasad and Tran (2012) in which workers can make investments in two types of human capital (general and firm-specific), and only when workers invest in both types of skills do firms promote them to challenging jobs featuring high returns to skills.
The main tradeoff in our model is between first-period incentives and second-period task assignments. A related tradeoff has been noted in the promotions literature. When firms use promotions both to create incentives and to achieve efficient job assignment, Baker, Jensen, and Murphy (1988) observed that it is generally not possible to perfectly achieve both objectives. For example, promoting the best-performing salesperson (as required for incentives) is unlikely to yield the best manager. Furthermore, the sequencing of the decisions of lower-level workers and the firm creates a time-inconsistency problem, as argued in Milgrom and Roberts (1988). Workers take actions to increase their promotion probabilities, but those investments are already water under the bridge and cannot be undone when the time comes for the firm to make a promotion decision. Workers suffer depressed incentives given their anticipation that the firm will make promotion decisions solely on the basis of efficiency in job assignments. The firm maximizes total profit by adhering to the ex ante optimal promotion rule (that reflects both incentives and job assignment considerations), but in the absence of commitment the firm will deviate by following the ex post optimal rule that reflects only assignment considerations. As shown in Waldman (2003), the common practice of favoring internal candidates (as opposed to external recruits) for promotion can be understood as a means by which the firm commits to the ex ante optimal promotion rule. In the preceding models, the worker’s investment is unidimensional so that the distortion caused by time inconsistency leads to underinvestment. In contrast, in our model the worker’s investment is two dimensional, so that the relevant distortion arising from time inconsistency is across tasks within the low-level job. Later in the analysis we discuss a possible solution to the commitment problem, in which the firm prefers not to renege on its promotion rule ex post so as to avoid creating disgruntled workers who might retaliate against their employer in ways that are hard to monitor.

Finally, the paper is closely related to the multitasking literature (e.g. Holmström and Milgrom 1991, Baker 1992, 2002). This literature emphasizes the problems that pay-for-performance schemes entail when workers engage in multiple tasks and some tasks are easier to measure than others. Under a pay-for-performance scheme workers have an incentive to focus on those tasks that are captured by the performance measure and, thus, rewarded by the incentive contract, while neglecting the remaining tasks. As a consequence, firms may set rather weak incentives or may abstain from using pay-for-performance

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5 Schnedler (2008) shows that this result is not necessarily true if workers find some tasks easier to accomplish than others.
schemes altogether to induce workers to focus on all of their tasks. In the current paper it is not a (distorted) performance measure, but the endogenously chosen promotion rule that induces workers to focus more strongly on one of the tasks. In addition, addressing the multitasking problem by using a more balanced promotion rule leads to a cost in terms of a worse assignment of workers to jobs. Such assignment problems are typically absent in the existing literature on multitasking.

2 Model description

In a two-period setup, consider a firm with a two-level job hierarchy. There is one job at each level, where job 1 is the low-level job, and job 2 is the high-level job. Each job consists of two tasks, $a$ and $b$, that differ in relative importance across jobs. The employment relationships begin when the risk-neutral firm owner (henceforth, the firm) hires two risk-neutral workers into job 1 at the start of the first period. Let $t$, $k$, $j$, and $i$ index periods, jobs, tasks, and workers, respectively. Let $a_{ij}$ denote worker $i$’s ability on task $j$, which at the outset is unknown to all parties. Suppose $a_{ij}$ is an i.i.d. random variable that is distributed on $(-\infty, \infty)$ with continuously differentiable pdf $g$ (that has full support) and cdf $G$, and define $\bar{a} := E[a_{ij}]$. Let $e_{ij} \geq 0$ denote worker $i$’s effort on task $j$ in period $t$, which is unobservable to the firm, and let $q > 0$ denote the productivity of that effort. Worker $i$’s effort cost is given by $c_i = \frac{1}{2} \sum_{j,t} (e_{ij})^2$ (with $c > 0$). When employed in job $k$ in period $t$, worker $i$ produces output $y_{ik} = \tau_k (a_{ia} + qe_{ia}^t) + (1 - \tau_k) (a_{ib} + qe_{ib}^t)$ for the firm, where $\tau_k \in (0, 1) (1 - \tau_k)$ measures the importance of task $a$ (task $b$) in job $k$. Without loss of generality,
we assume $\tau_1 < \tau_2$, so that task $a$ receives relatively more emphasis in the high-level job. For example, task $a$ might represent leadership activities, whereas task $b$ might represent "actual work", i.e. direct production.

At the end of the first period, the firm observes relative performance signals $s_j = a_{1j} + e_{1j}^1 - a_{2j} - e_{2j}^1$. These signals are nonverifiable to third parties and thus cannot be used in a formal incentive contract. On the basis of the signals, the firm promotes one of the workers to job 2 at the start of the second period and retains the other worker in job 1. Letting $s_{ij} := a_{ij} + e_{ij}^1$, the firm promotes worker 1 if and only if $\theta s_a + (1 - \theta) s_b > 0 \Leftrightarrow \theta s_{1a} + (1 - \theta) s_{1b} > \theta s_{2a} + (1 - \theta) s_{2b}$. For now we assume that the firm is able to commit to a promotion rule, i.e. to a specific choice of $\theta \in (0, 1]$ at the beginning of period 1, and that the workers observe this when making first-period effort choices. Later we discuss the possibility of a commitment problem and how it might be resolved.

We start with the simplifying assumption that wages are exogenously given, where $w_1$ is the first-period wage, $w_{2H}$ is the second-period wage of the promoted worker, $w_{2L}$ is the second-period wage of the worker who is not promoted, and $(w_{2H} > w_{2L})$. Workers’ reservation values are normalized to zero. Later in the analysis we allow wages to be endogenously chosen by the firm. Finally, workers and the firm discount future payoffs at factors $\delta_W \in (0, 1)$ and $\delta_F \in (0, 1)$, respectively.

3 Model solution

We solve the model by backward induction. Define $S_i := \theta s_{ia} + (1 - \theta) s_{ib}$, $i = 1, 2$. As described in the following proposition, the solution for period 2 is strikingly simple.\footnote{Since our primary goal is to analyze whether the firm prefers to set $\theta = 1$ or $\theta < 1$, it does not really matter whether the possibility of setting $\theta = 0$ is included or excluded. Given that we must sometimes divide by $\theta$ in the formal analysis, the case of $\theta = 0$ would need to be analyzed separately. Since this would lengthen the analysis without providing any new insights, we have excluded the possibility of setting $\theta = 0$.}

**Proposition 1** In period 2, the workers choose $e_{ij} = 0 \forall i, j$. The firm’s expected second-period profit is given by

\[
\pi_2 = E[a_{1b} | S_1 > S_2] + \tau_2 (E[a_{1a} | S_1 > S_2] - E[a_{1b} | S_1 > S_2])
+ E[a_{2b} | S_1 > S_2] + \tau_1 (E[a_{2a} | S_1 > S_2] - E[a_{2b} | S_1 > S_2]) - w_{2H} - w_{2L}.
\]

\footnote{All proofs are in the appendix.}
Since the game ends after period 2, the workers choose the minimum effort level. Thus, only the workers’ abilities affect second-period profit. We have assumed that the relative importance of task $a$ versus $b$ is higher in job 2 than in job 1. Therefore, it is intuitive that ex post the firm owner would benefit from promoting the worker who has the higher ability on task $a$. We formalize this intuition later by demonstrating that, beginning at $\theta = 1$ expected second-period profit decreases as $\theta$ is lowered. Turning now to period 1, worker $i$ chooses first-period efforts to maximize

$$EU_i = w_1 + \delta_W w_{2L} - \frac{c}{2} \left( (e_{ia}^1)^2 + (e_{ib}^1)^2 \right) + \Pr \{ S_i \geq S_l \} \delta_W (w_{2H} - w_{2L}),$$

with $l = 1, 2, l \neq i$. The probability that worker $i$ gets promoted can be restated as

$$\Pr \{ S_i \geq S_l \} = \Pr \left\{ \theta \left( a_{ia} + e_{ia}^1 \right) + (1 - \theta) \left( a_{ib} + e_{ib}^1 \right) > \theta \left( a_{ia} + e_{ia}^1 \right) + (1 - \theta) \left( a_{ib} + e_{ib}^1 \right) \right\}$$

$$= \Pr \left\{ a_{ia} + e_{ia}^1 - e_{ia}^1 + \frac{(1 - \theta)}{\theta} \left( a_{ib} + e_{ib}^1 - a_{ib} - e_{ib}^1 \right) > a_{ia} \right\}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G \left( a_{ia} + e_{ia}^1 - e_{ia}^1 + \frac{(1 - \theta)}{\theta} \left( a_{ib} + e_{ib}^1 - a_{ib} - e_{ib}^1 \right) \right) g(a_{ia}) da_{ia} g(a_{ib}) da_{ib}.$$
Optimal efforts of worker $i$ are characterized by the following first-order conditions:

$$
\frac{\partial EU_i}{\partial e^1_{ia}} = -c^1_{ia} + \frac{\partial \Pr \{S_i \geq S_l\}}{\partial e^1_{ia}} \delta_W (w_{2H} - w_{2L}) = 0
\iff e^1_{ia} = \frac{\partial \Pr \{S_i \geq S_l\} \delta_W (w_{2H} - w_{2L})}{c},
$$

$$
\frac{\partial EU_i}{\partial e^1_{ib}} = -c^1_{ib} + \frac{\partial \Pr \{S_i \geq S_l\}}{\partial e^1_{ib}} \delta_W (w_{2H} - w_{2L}) = 0
\iff e^1_{ib} = \frac{\partial \Pr \{S_i \geq S_l\} \delta_W (w_{2H} - w_{2L})}{c}.
$$

Note that

$$
\frac{\partial \Pr \{S_i \geq S_l\}}{\partial e^1_{ia}} =
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g \left( a_{ia} + e^1_{ia} - e^1_{la} + \frac{(1-\theta)}{\theta} (a_{ib} + e^1_{ib} - a_{lb} - e^1_{lb}) \right) \cdot g(a_{ia}) da_{ia} g(a_{ib}) da_{ib}
$$

and

$$
\frac{\partial \Pr \{S_i \geq S_l\}}{\partial e^1_{ib}} =
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g \left( a_{ia} + e^1_{ia} - e^1_{la} + \frac{(1-\theta)}{\theta} (a_{ib} + e^1_{ib} - a_{lb} - e^1_{lb}) \right) \cdot g(a_{ia}) da_{ia} g(a_{ib}) da_{ib}.
$$

\footnote{Of course, such a first-order approach is only valid if workers’ objective functions are strictly concave. As usual in tournament models, satisfying this condition requires additional assumptions. In particular, the objective function is ensured to be strictly concave for a sufficiently large $c$ (see the Appendix for a proof). In the following, we assume that $c$ is large enough that optimal efforts can be characterized by the first-order conditions.}
In the appendix, we show that the equilibrium is symmetric, with both workers choosing the same effort on task \( j \). Then the preceding derivatives become

\[
\frac{\partial \Pr \{ S_i \geq S_l \}}{\partial e_{ia}^j} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g \left( a_{ia} + \frac{(1 - \theta)}{\theta} (a_{ib} - a_{lb}) \right) g (a_{ia}) da_{ia} g (a_{ib}) da_{ib} g (a_{lb}) da_{lb}
\]

and

\[
\frac{\partial \Pr \{ S_i \geq S_l \}}{\partial e_{ib}^j} = \frac{1 - \theta}{\theta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g \left( a_{ia} + \frac{(1 - \theta)}{\theta} (a_{ib} - a_{lb}) \right) g (a_{ia}) da_{ia} g (a_{ib}) da_{ib} g (a_{lb}) da_{lb}.
\]

The following proposition characterizes optimal first-period efforts:

**Proposition 2** Optimal efforts satisfy \( e_{ia}^1 \geq e_{ib}^1 \) if and only if \( \theta \geq 0.5 \).

The workers exert effort to positively influence the signal realizations and, thus, to increase the probability of being promoted. Recall that \( \theta \) determines the importance of the performance signal on task \( a \) for the promotion decision. If \( \theta = 0.5 \), both performance signals have the same effect on the promotion decision, so that workers find it optimal to allocate the same amount of effort to each task. If instead \( \theta > 0.5 \), the performance signal on task \( a \) is more important for the promotion decision, and workers shift effort from task \( b \) to task \( a \). Similarly, if \( \theta < 0.5 \), task \( b \) is the more important one so that workers choose higher effort on task \( b \).

Given the preceding considerations, expected first-period profit can be written as

\[
\pi_1 = 2 \left( \alpha (1 + q e_{ia}^1) + (1 - \tau_1) (a + q e_{ib}^1) - w_1 \right)
\]

\[
= 2a - 2w_1 + 2q \left( \tau_1 e_{ia}^1 + (1 - \tau_1) e_{ib}^1 \right)
\]

\[
= 2a - 2w_1 + 2q \frac{\delta W (w_2 H - w_2 L)}{c} X (\theta),
\]
with
\[
X(\theta) := \frac{(1 - \theta + \tau_1 (2\theta - 1))}{\theta} \cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g \left( a_{ia} + \frac{(1 - \theta)}{\theta} (a_{ib} - a_{lb}) \right) g (a_{ia}) da_{ia} g (a_{ib}) da_{ib}.
\]

The following partial derivative describes how this profit changes with \( \theta \)
\[
\frac{\partial \pi_1}{\partial \theta} = 2q \frac{\delta W (w_2H - w_2L)}{c} X'(\theta)
\]
\[
= 2q \frac{\delta W (w_2H - w_2L)}{c} \cdot \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g \left( a_{ia} + \frac{(1 - \theta)}{\theta} (a_{ib} - a_{lb}) \right) \cdot \right.
\]
\[
\left. \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g' \left( a_{ia} + \frac{(1 - \theta)}{\theta} (a_{ib} - a_{lb}) \right) (a_{ib} - a_{lb}) \frac{1}{\eta^2}. \right)
\]

Evaluating this derivative as \( \theta \) approaches 1, we obtain the following lemma.

**Lemma 1** \( \lim_{\theta \to 1} \frac{\partial \pi_1}{\partial \theta} < 0 \).

The lemma states that first-period profit increases as \( \theta \) decreases from 1. Lowering \( \theta \) increases expected first-period profit by changing the workers’ incentives to manipulate the performance signals. If \( \theta \) is lowered, the workers are less inclined to shift effort from task \( b \) to task \( a \), which in turn boosts the workers’ productivities. However, this first-period incentive effect must be traded off against a second-period task assignment effect that is countervailing, as shown in the next lemma.

**Lemma 2** \( \lim_{\theta \to 1} \frac{\partial \pi_2}{\partial \theta} > 0 \).

The lemma states that lowering \( \theta \) from 1 reduces the firm’s second-period profit. This is intuitive. If \( \theta \) is maximized, the workers are assigned to the jobs that best match their abilities.

The following proposition shows that there are cases in which the incentive effect dominates the assignment effect. In such cases the firm finds it optimal
to choose $\theta < 1$. This means that in the interests of strengthening first-period incentives the firm chooses to take into account performance signals on both tasks when making the promotion decision, even though the promoted worker’s skill on task $a$ is the more relevant one.

**Proposition 3** There exists a cutoff value $\tilde{q}(c, \delta_F, \delta_W)$ such that the firm chooses $\theta < 1$ iff $q \geq \tilde{q}(c, \delta_F, \delta_W)$. The cutoff value satisfies $\frac{\partial \tilde{q}(c, \delta_F, \delta_W)}{\partial c} > 0$, $\frac{\partial \tilde{q}(c, \delta_F, \delta_W)}{\partial \delta_F} > 0$ and $\frac{\partial \tilde{q}(c, \delta_F, \delta_W)}{\partial \delta_W} < 0$.

To understand the logic for Proposition 3, it is helpful to recall that $q$ represents the productivity of the worker’s effort and also that second-period equilibrium efforts are zero, so that the only role of $q$ in equilibrium is amplifying the effect of first-period efforts on first-period profit. The first sentence of Proposition 3 says that when $q$ is sufficiently high, the firm decides to consider performance signals on both tasks in the promotion decision. The logic for this result is as follows. From Lemma 1 and its proof, the benefit to the firm of considering both performance signals for the promotion decision is an increase in first-period profit that arises because the workers’ weighted total effort on the two tasks increases. Such increases in effort increase first-period profit to a greater extent the larger is $q$. Hence, for sufficiently large $q$, the firm finds it beneficial to consider performance signals on both tasks in the promotion decision. This establishes a threshold for $q$.

The second sentence of Proposition 3 establishes three comparative statics results concerning how the threshold for $q$ varies with the worker’s effort cost and with the discount factors of the worker and firm.$^{13}$ Note that an increase in $c$ increases the workers’ total and marginal costs of exerting effort, so that the firm finds it harder to induce effort. From Lemma 1 and its proof, the way that the firm induces first-period effort is by committing to use both performance signals when making the promotion decision (i.e. reducing $\theta$), and when $c$ is higher a reduction in $\theta$ affects first-period effort to a lesser extent. This means that when $c$ is high the first-period incentive effect shrinks in importance relative to the second-period assignment effect. In this situation the attractiveness to the firm of considering both performance signals in the promotion decision is diminished, and the firm would need to see an even greater productivity of effort (i.e. value of $q$) before deciding to use both signals when deciding about promotion.

$^{13}$Section 4.3 contains additional comparative statics results.
From Lemma 1 and its proof, when the workers’ discount factor is high first-period effort becomes more responsive to a reduction of $\theta$ below 1. The reason the workers invest first-period effort is to become promoted so that the higher second-period wage can be collected. The workers’ discount factor captures the importance to the workers of receiving this higher second-period wage, and when the discount factor is zero the workers exert no effort on either task. The more the workers care about the future the easier it is for the firm to create first-period incentives via committing to use both performance signals in the promotion decision. In this situation the attractiveness to the firm of using both signals when making the promotion decision is increased, and the firm would be willing to commit to it even when the productivity of effort is modest. Hence, the threshold for $q$ diminishes as the workers’ discount factor increases.

The intuition for why the threshold for $q$ is increasing in the firm’s discount factor follows from the observation that an increase in the discount factor increases the contribution of second-period profit (relative to first-period profit) in the firm’s total profit expression. Recall that when the firm commits to consider both performance signals when deciding about promotion (i.e. decreases $\theta$) this increases first-period profit at the expense of decreasing second-period profit. This expense of decreased second-period profit is more pronounced the larger is the firm’s discount factor; for example, in the extreme case in which the firm has a discount factor of zero, this expense would disappear entirely.

In short, when the firm’s discount factor is high and the firm decreases $\theta$, the benefit of the first-period incentive effect shrinks in importance relative to the cost of the second-period assignment effect. In this situation the attractiveness to the firm of considering both performance signals in the promotion decision is diminished, and the firm would need to see an even greater productivity of effort (i.e. value of $q$) before being willing to take both signals into account when making the promotion decision.

As noted in the introduction, Putt’s Law states that promoted workers ”manage what they do not understand” whereas non-promoted workers ”understand what they do not manage.” The ”Law” was first articulated in 1981, and a closely related formulation called the ”Dilbert Principle” appeared in the 1990s. 14 A

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14 This was a satirical observation by the Dilbert cartoonist Scott Adams, stating that companies tend to systematically promote their least competent employees to management so as to minimize the harm they can do. In a Dilbert comic strip on February 5, 1995, the character Dogbert says ”leadership is nature’s way of removing morons from the productive flow.” In the words of Scott Adams, ”I wrote The Dilbert Principle around the concept that in many cases the least competent, least smart people are promoted, simply because they’re the ones you don’t want doing actual work. You want them ordering the doughnuts and yelling
corollary to Putt’s Law, sometimes referred to either as "Putt’s Corollary" or as the "First Corollary to Putt’s Law" further states that every technical hierarchy eventually develops a competence inversion, meaning that over time the incompetent workers get promoted to "manage what they do not understand" and the competent ones are left in lower-level jobs to "understand what they do not manage". The fact that our model is characterized by multiple tasks in each job allows us to explain how Putt’s Law can be understood as a natural consequence of the firm’s optimizing behavior. Before presenting the formal argument, we consider the question of whether the "Law" has any practical relevance. One might be tempted to dismiss the "Law" as simply reflecting the bloated senses of self-esteem experienced by a subset of workers who were justifiably denied promotions on performance grounds. But even a cursory review of the anecdotal evidence available online reveals there is more to the notion of "too good to get promoted" than can be explained solely by the delusions of disgruntled workers. Indeed, employers sometimes admit outright to withholding promotions because a worker’s performance was too high. For example, consider the following account on the "Yahoo! Answers" website, posted under the heading "I’m too good at my job to get promoted?"

I work in the IT industry of a nation-wide corporate medical provider. I have also worked in smaller call centers. My resume is impressive, and I have management and shift lead experience. I left my last job after 2 years as shift lead, because they would not hire me to the open call center manager position. When I spoke with the owner, he told me it was because I was "too valuable on the phones." So now, at my new job, a network engineer opportunity opened, and that’s what my degree is in. When I applied for the position, I spoke with the help desk manager and he told me that he’d love to help me progress in my career, but my ticket-completed volume is too high for him to move me off of the phones, and that I was too good to let move right now. I can’t get a manager position anywhere else, because I can never get a manager position where I’m at...what am I doing wrong by being so good at my job? Suggestions on what I

at people for not doing their assignments - you know, the easy work. Your heart surgeons and your computer programmers - your smart people - aren’t in management. That principle was literally happening everywhere.”

A similar competence inversion is predicted by the Peter Principle but for a different reason. The Peter Principle states that workers are promoted because they are competent in the tasks at hand, and that process continues until they become incompetent because they eventually encounter tasks they cannot handle, whereas Putt’s Law and the Dilbert Principle state that workers get promoted precisely because they are incompetent (at least in the tasks emphasized in the low-level job, which are sometimes the most important productive tasks).
could do to improve my career path?

Such examples suggest that Putt’s Law is relevant and, importantly, that instances of the Law cannot always be understood as mistakes on the part of firms. Sometimes firms purposefully hold back their highest performers, because the opportunity cost of promoting them is too high. Turning now to the formal argument, suppose that task $a$ represents management activities whereas task $b$ is a production task that some workers might view as "real work". If ability on task $b$ is higher for the non-promoted worker than for the promoted worker, this pattern is consistent with Putt’s Law. The following argument establishes the likelihood of such a pattern. Define $A := a_{1a} - a_{2a}$ and $B := a_{1b} - a_{2b}$. Note that $A$ and $B$ are i.i.d. and symmetric around zero, and denote the pdf by $h$ and the cdf by $H$. Because both workers choose the same efforts, our aim is to determine

$$Z(\theta) := \Pr\{ B < 0 | \theta A + (1 - \theta) B > 0 \}$$

and to analyze how this probability depends on $\theta$.

For $\theta < 1$, we can restate the preceding probability as

$$Z(\theta) = \Pr\left\{ B < 0 | B > -\frac{\theta}{1 - \theta} A \right\}$$

$$= \Pr\left\{ A > 0 \land B < 0 | B > -\frac{\theta}{1 - \theta} A \right\} + \Pr\left\{ A \leq 0 \land B < 0 | B > -\frac{\theta}{1 - \theta} A \right\}$$

$$= \Pr\left\{ A > 0 \land B < 0 | B > -\frac{\theta}{1 - \theta} A \right\}.$$ 

Using Bayes’ rule, $Z(\theta)$ becomes

$$Z(\theta) = \frac{\Pr\left\{ A > 0 \land -\frac{\theta}{1 - \theta} A < B < 0 \right\}}{\Pr\left\{ B > -\frac{\theta}{1 - \theta} A \right\}},$$

which in turn can be rewritten as

$$Z(\theta) = \frac{\int_{0}^{+\infty} \int_{-\frac{\theta}{1 - \theta} A}^{0} h(B) dH(A) dA}{\int_{-\infty}^{+\infty} \int_{-\frac{\theta}{1 - \theta} A}^{+\infty} h(B) dH(A) dA}
= \frac{\int_{0}^{+\infty} \left( 0.5 - H\left( -\frac{\theta}{1 - \theta} A \right) \right) h(A) dA}{\int_{-\infty}^{+\infty} \left( 1 - H\left( -\frac{\theta}{1 - \theta} A \right) \right) h(A) dA}.$$
\[
R_0 H \begin{pmatrix} A \\ h(A) dA \\ 0 \end{pmatrix} = R_0 H \begin{pmatrix} A \\ h(A) dA \end{pmatrix} + R_0 H \begin{pmatrix} A \\ h(A) dA \end{pmatrix}.
\]

Note that we can rewrite \( R_0 H \begin{pmatrix} A \\ h(A) dA \end{pmatrix} \) as
\( R_0 H \begin{pmatrix} A \\ h(A) dA \end{pmatrix} \), which by the symmetry of the distribution is equal to
\( R_0 H \begin{pmatrix} A \\ h(A) dA \end{pmatrix} \).

Then \( Z(\theta) \) simplifies to
\[
Z(\theta) = \frac{\int_0^{+\infty} H \left( \frac{\theta}{1-\theta} A \right) h(A) dA - 0.25}{\int_0^{+\infty} H \left( \frac{\theta}{1-\theta} A \right) h(A) dA}.
\]

Hence, we obtain
\[
Z'(\theta) = 2 \int_0^{+\infty} h \left( \frac{\theta}{1-\theta} A \right) \frac{1}{(1-\theta)^2} A h(A) dA,
\]
which is strictly positive. Finally, we can show that \( Z(\theta) \) is continuous at \( \theta = 1 \) since
\[
\lim_{\theta \to 1} Z(\theta) = 2 \int_0^{+\infty} h(A) dA - 0.5 = 0.5 = Z(1).
\]

The following proposition summarizes the results:

**Proposition 4** If \( \theta > 0 \), the non-promoted worker has a higher ability on task \( b \) than the promoted worker with positive probability (and the probability is 0.5 when \( \theta = 1 \)). This ability pattern becomes more likely as \( \theta \) increases.

Proposition 4 states that an ability pattern consistent with Putt’s Law is observed with positive probability. That is, those workers who are the best performers in the job at hand end up "understanding what they do not manage" whereas those whose productivity in the low-level job is less impressive get promoted to "manage what they do not understand." If \( \theta = 1 \), the non-promoted worker has a higher ability on task \( b \) than the promoted worker with probability 0.5. Hence, an ability pattern that is consistent with Putt’s Law is most likely to be observed if strategic shirking is relatively unimportant so that the firm finds
it optimal to choose $\theta = 1$. Firms that are sensitive to the strategic shirking problem can mitigate it by requiring more balance between tasks in the stated promotion rule, so that workers who excel in the task that is most productive in the low-level job are less disadvantaged when it comes to promotion decisions. But as we discuss in the next section, firms may vary in the extent to which they can credibly commit to a stated promotion rule, and consequently strategic shirking in promotion tournaments (and Putt’s Law) may vary in intensity across production settings.

It is worth emphasizing that in our model Putt’s Law does not arise as a consequence of *ex post* mistakes by the firm but rather as a purposeful choice as part of an optimizing strategy in the case of complete certainty at the moment second-period job assignments are made. An alternative way to generate Putt’s Law would be to assume that the employer can only observe error-laden measures of task-specific relative performance. In that case, workers who get particularly lucky (meaning that measurement error leads the employer to think they are better than they really are) tend to get promoted in equilibrium. However, given that luck is not persistent, such workers will tend not to perform as well once promoted.\(^\text{16}\) Thus, with positive probability a pattern of job assignments consistent with Putt’s Law occurs. There is an important difference between this alternative explanation for Putt’s Law and our explanation. In the case of the alternative explanation it would be expected that firms might want to correct mistaken job assignments *ex post* via demotions when the promoted workers’ poor performances become apparent. But demotions tend to be pretty rare in practice.\(^\text{17}\) In contrast, in our model, given that there is no uncertainty in the firm’s observations of relative performance, the instances of Putt’s Law that occur with positive probability are not mistakes that the firm would ever wish to correct *ex post*, and hence there would be no need for demotions. Our model can explain Putt’s Law as rational behavior even with complete information, and since no mistakes are made, the situation in equilibrium can be expected to persist with no tendency for demotions or firings.

\(^{16}\)See Lazear (2004) for a similar argument concerning the Peter Principle.

\(^{17}\)See, for example, the seminal empirical studies on internal labor markets by Baker, Gibbs, and Holmström (1994a,b).

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4 Extensions

In this section we consider some extensions of the model. First, we consider the case in which wage levels are endogenously chosen by the firm rather than exogenously fixed as in the preceding section. Second, we discuss a possible solution to the potential commitment problem the firm may face in stating the promotion rule upfront. Third, we consider how our results vary with the degree to which jobs are similar across hierarchical levels (in the productivity weights the jobs attach to each task). Fourth, we consider the firm’s optimal promotion policy if a pay-for-performance scheme based on an additional performance signal were feasible.

4.1 Endogenous prizes

In this subsection, we analyze whether the previous results continue to hold if the firm chooses optimal prizes. Following the seminal paper by Lazear and Rosen (1981), we assume that the firm can commit to the payment of the prizes at the beginning of period 1. We further assume that the firm has complete bargaining power and makes a take-it-or-leave-it-offer to both workers. In doing so, it must account for the workers’ participation constraints in the first and second period. We further simplify the situation by assuming \( \delta_F = \delta_W =: \delta \).

The firm’s maximization problem is then given by

\[
\max_{w_1, w_{2H}, w_{2L}, \theta} 2\theta + 2 \left( \tau_1 q e_{ia}^1 + (1 - \tau_1) q e_{ib}^1 \right) - 2w_1 \\
+ \delta \left( E \left[ \frac{\tau_2 q e_{ia}^1 + (1 - \tau_2) q e_{ib}^1}{\theta a_{ia} + (1 - \theta) a_{ib} + \tau_1 a_{2a}} + (1 - \tau_1) a_{2b} | \theta a_{ia} + (1 - \theta) a_{ib} > \theta a_{2a} + (1 - \theta) a_{2b} \right] - w_{2H} - w_{2L} \right) \\
\text{s.t. } e_{ij}^1 = \frac{\partial \Pr \{ S_i \geq S_j \} \delta (w_{2H} - w_{2L})}{c} \text{ for } j = a, b \quad (IC), \\
w_1 - \frac{c}{2} \left( (e_{ia}^1)^2 + (e_{ib}^1)^2 \right) + \frac{\delta w_{2H} + w_{2L}}{2} \geq 0 \quad (PC_1), \\
w_{2H}, w_{2L} \geq 0 \quad (PC_2).
\]

It is straightforward to see that the firm always sets \( w_1 \) such that \( (PC_1) \) binds, i.e. the firm leaves no rents to the workers. The firm’s maximization problem
can thus be rewritten as

$$\begin{align*}
\max_{w_{2H}, w_{2L}, \theta} & \quad 2\bar{a} + 2\left(\tau_1 q e_{ia}^1 + (1 - \tau_1) q e_{ib}^1\right) - c \left(\left(e_{ia}^1\right)^2 + \left(e_{ib}^1\right)^2\right) + \delta \left(\mathbb{E} \left[\left(\tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} \mid \theta a_{1a} + (1 - \theta) a_{1b} > \theta a_{2a} + (1 - \theta) a_{2b}\right)\right]\right) \\
\text{s.t.} & \quad e_{ij}^1 = \frac{\partial \Pr \{S_i \geq S_j\} \delta (w_{2H} - w_{2L})}{\delta e_{ij}} \text{ for } j = a, b \quad (IC), \\
& \quad w_{2H}, w_{2L} \geq 0 \quad (PC_2).
\end{align*}$$

We observe that the firm’s expected total profit depends only on the difference in prizes, $w_{2H} - w_{2L}$, but not on the level of prizes. Without loss of generality, we can therefore set $w_{2L} = 0$. We begin by assuming that the firm chooses $\theta = 1$, in which case optimal effort on task $b$ is always equal to zero (since $\frac{\partial \Pr \{S_i \geq S_j\}}{\delta e_{ia}^1} = 0$).

Because the workers receive no rent, the firm receives the complete surplus that the workers produce. Hence, the firm induces the (efficient) effort level on task $a$ that maximizes the surplus. In particular, the firm chooses $w_{2H}$ to maximize

$$2\tau_1 q e_{ia}^1 - c \left(\frac{e_{ia}^1}{c}\right)^2,$$

which leads to an optimal effort of

$$e_{ia}^1 = \frac{\tau_1 q}{c},$$

and a corresponding winner prize of

$$w_{2H} = \frac{\tau_1 q}{\delta \int \int \int \int (g(a_{ia}))^2 da_{ia} g(a_{ib}) da_{ib} g(a_{ib}) da_{ib}}.$$

Suppose now that the firm marginally reduces $\theta$ below 1, but changes $w_{2H}$ such that the workers still find it in their interest to choose effort $e_{ia}^1 = \frac{\tau_1 q}{c}$ on task $a$. This requires a post-promotion wage of

$$w_{2H} = \frac{\tau_1 q}{\delta \int \int \int \int g(a_{ia} + \frac{(1-\theta)}{\theta} (a_{ib} - a_{ib})) g(a_{ia}) da_{ia} g(a_{ib}) da_{ib} g(a_{ib}) da_{ib}}.$$
in which case effort on task \( b \) would be given by
\[
e^{1}_{ib} = \frac{1 - \theta q\tau_1}{c}.
\]

Note that the additional profit from the positive effort on task \( b \) is
\[
2(1 - \tau_1) q e^{1}_{ib} - c(e^{1}_{ib})^2 =
\]
\[
\frac{q^2}{\theta^2}\tau_1 (1 - \theta) (2\theta - \tau_1 (1 + \theta)).
\]

For \( \theta \) sufficiently close to 1, the expression is strictly positive, because
\[
\lim_{\theta \to 1} (2\theta - \tau_1 (1 + \theta)) = 2 (1 - \tau_1) > 0.
\]
Furthermore, if \( q \) is sufficiently high, this additional profit always overcompensates the second-period loss in profit due to the reduction in \( \theta \). The following proposition is therefore immediate.

**Proposition 5** In the model with endogenous prizes, the firm always finds it optimal to set \( \theta < 1 \) if \( q \) is sufficiently high.

Proposition 5 is very intuitive. By changing the size of the prizes, the firm affects workers’ incentives to put forth effort in period 1. The prize structure, however, does not affect the allocation of efforts across tasks. To see this, notice that \( e^{1}_{ia} \) is given by
\[
e^{1}_{ia} = \frac{\frac{\partial \Pr\{S_i > S_l\}}{\partial e^{1}_{ia}}}{\frac{\partial \Pr\{S_i > S_l\}}{\partial e^{1}_{ib}}},
\]
which is independent of \( w_1, w_{2H} \) and \( w_{2L} \). Hence, even with optimally chosen prizes the firm sometimes finds it useful to reduce \( \theta \) below 1 to achieve a more desirable allocation of efforts across tasks.

### 4.2 Commitment to the stated promotion rule

We have assumed that performance signals are nonverifiable, which seems natural in many economic settings. For example, promotions are often based on subjective assessments of worker performance, which are difficult to verify in court. The problem then is that the firm may want to lie about the signal realizations and deviate from the stated promotion rule. Suppose, for example, that \textit{ex post} the firm’s profit would be maximized if worker 1 was promoted, but
that worker 2 is promoted when the firm follows the specified promotion rule (this, of course, requires a choice of \( \theta < 1 \)). Then the firm could lie about the signal realizations, thereby increasing worker 1’s performance relative to that of worker 2 so that worker 1 gets promoted.

In the real world, what prevents the firm from lying about the signal realizations (and thus reneging on an implicit contract with the workers) even when there is nonverifiability is a fear of getting punished by disgruntled workers. Even if workers have no direct recourse through a third party like a court, they are still in a position to impose costs on the firm. When it comes to promotions, if contest rules are stated, and if the parties all see that worker 2 performs better but worker 1 gets the promotion, worker 2 is disgruntled and may take steps to sabotage the firm. On the other hand, if 1 performs better than 2 and gets the promotion, 2 is not disgruntled because he knew the rules and knows he lost fair and square. Basically in an ongoing employment relationship, it is costly to the firm to have disgruntled workers around.

One way to incorporate this idea in the model is the following: Suppose that \( e_{ij}^t \) is no longer bounded below by 0 but can assume any value on the real line. The workers’ expected utility function is the same as before except that a new term is subtracted from it, namely \( \lambda I_i^t e_{ij}^t \), where \( \lambda > 0 \), and \( I_i^t \) is an indicator function equaling 1 if worker \( i \), in period \( t \), failed to get promoted even though \( S_i > S_l \). By this definition, \( I_i^1 = 0 \) in period 1, so that the utility function is the same as in the basic model, and therefore negative choices of \( e_{ij}^1 \) are strictly dominated by non-negative ones. But in period 2, if the indicator function turns on because the firm lies about the signal realizations and deviates from the stated promotion rule, worker \( i \) becomes disgruntled and retaliates. So \( \lambda \) can be interpreted as the worker’s “marginal propensity to sabotage the firm” after being unfairly denied promotion. It may now be optimal for worker \( i \) to choose \( e_{ij}^2 < 0 \), hurting the firm’s profit. For sufficiently high \( \lambda \), the firm should never find it optimal to lie about the signal realization in equilibrium, even with nonverifiability.

An argument might be made that if the worker retaliates against the firm, he will get fired. There are at least two responses to this argument. One is that, in a two-period model, even if the worker is fired, the damage is already done and the firm has suffered costs. Another response is that the sabotage is frequently not visible to the firm. It is done in subtle ways. In the firm’s eyes, \( e_{ij}^2 < 0 \) might very well look the same as \( e_{ij}^2 = 0 \). Workers can impose costs in a variety of ways with virtually no risk of getting caught, and in fact the ability
to do so is what makes sabotage possible.

4.3 Similarity of tasks across hierarchical levels

Hierarchies can be classified according to the “degree to which tasks are similar across hierarchical levels”. Compare two hierarchies. One is in academia, with assistant professors in job 1 and associate professors in job 2, where the tasks are essentially identical at both levels. Another is computer programmers at the bottom and managers at the top, so that tasks differ substantially across levels. Our model captures task variety by means of the difference $\tau_2 - \tau_1$. When this difference increases, tasks across hierarchical levels differ to a larger extent. In this section, we analyze how $\tau_2 - \tau_1$ affects decisions in our model. We start by varying $\tau_2$ while keeping $\tau_1$ constant.

It is straightforward to see that workers’ efforts do not depend on $\tau_2$, so we can focus on the firm’s choice of $\theta$. It is also straightforward to see that $\pi_1$ is independent of $\tau_2$, and hence $\frac{\partial \pi_1}{\partial \tau_2} = 0$. However, $\tau_2$ has an impact on second-period profit and, in particular, on the marginal effect of an increase in $\theta$ on second-period profit. In the proof of Lemma 2 we have demonstrated that $\frac{\partial}{\partial \tau_2} (\lim_{\theta \to 1} \frac{\partial \pi_2}{\partial \theta})$ is strictly positive. This means that the firm is more inclined to choose $\theta = 1$ if $\tau_2$ increases and tasks across hierarchical levels differ more strongly. Put differently, if tasks across hierarchical levels differ significantly, the firm is more likely to promote a "specialist" who has demonstrated high ability on task $a$ (even if his ability on task $b$ is rather low).

The difference $\tau_2 - \tau_1$ would also be increased if $\tau_1$ were lowered while keeping $\tau_2$ constant. Changing $\tau_1$ leads to a more complex analysis, because $\tau_1$ affects both first-period and second-period profit. In the proof of Lemma 1 we have shown that

$$
\lim_{\theta \to 1} \frac{\partial \pi_1}{\partial \theta} = -2q \frac{\delta_{2H} (w_{2H} - w_{2L})}{c}
$$

$$
\left( (1 - \tau_1) \int\int\int g(a_{ia}) g(a_{ib}) da_{ia} g(a_{ib}) da_{ib} g(a_{ib}) da_{ib} \right).
$$
from which we obtain

\[
\frac{\partial}{\partial \tau_1} \left( \lim_{\theta \to 1} \frac{\partial \pi_1}{\partial \theta} \right) =
\]

\[
2q \frac{\delta_W (w_{2H} - w_{2L})}{c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(a_{ia}) g(a_{ia}) da_{ia} g(a_{ib}) da_{ib} g(a_{ib}) da_{ib} > 0.
\]

Hence, if \( \tau_1 \) is lowered, \( \lim_{\theta \to 1} \frac{\partial \pi_1}{\partial \theta} \) becomes lower as well (i.e. more negative). This means that the firm becomes more inclined to set \( \theta \) below 1, as a lower value of \( \tau_1 \) implies that task \( b \) becomes relatively more important, so that the firm wants to increase workers’ effort on that task.

In the proof of Lemma 2 we have demonstrated that

\[
\lim_{\theta \to 1} \frac{\partial \pi_2}{\partial \theta} = -2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( (1 - \tau_2) a_{1b}^2 + (\tau_2 - \tau_1) a_{1b} a_{2b} - (1 - \tau_1) a_{2b}^2 \right) \cdot
g(a_{1b}) da_{1b} g(a_{2b}) da_{2b} g^2(a_{2a}) da_{2a}.
\]

Hence, we observe that

\[
\frac{\partial}{\partial \tau_1} \left( \lim_{\theta \to 1} \frac{\partial \pi_2}{\partial \theta} \right) =
\]

\[
-2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( a_{2b}^2 - a_{1b} a_{2b} \right) g(a_{1b}) da_{1b} g(a_{2b}) da_{2b} g^2(a_{2a}) da_{2a}
\]

\[
= -2 \int_{-\infty}^{\infty} \left( E[a_{2b}^2] - E[a_{1b} a_{2b}] \right) g^2(a_{2a}) da_{2a},
\]

which is strictly negative since \( E[a_{1b} a_{2b}] = E[a_{1b}] E[a_{2b}] = (E[a_{2b}])^2 < E[a_{2b}^2] \).

Thus, if \( \tau_1 \) is lowered, \( \lim_{\theta \to 1} \frac{\partial \pi_2}{\partial \theta} \) gets higher, which implies that the firm is more inclined to choose \( \theta = 1 \) and to promote a "specialist".

In summary, an increase in \( \tau_2 - \tau_1 \) has countervailing effects on the firm’s decision to set \( \theta = 1 \) or \( \theta < 1 \). A higher value for \( \tau_2 - \tau_1 \) (regardless of whether it results from higher \( \tau_2 \), lower \( \tau_1 \), or both) implies that it is more important for the firm to promote the right employee. That is, selection becomes more important since tasks across hierarchical levels differ more strongly. As a result, the firm is more inclined to choose \( \theta = 1 \), i.e. to promote a worker solely based on
his ability on task $a$. At the same time, there may be incentive considerations that act in the opposite direction. If $\tau_1$ decreases, the importance of task $b$ for performance in job $1$ increases. The firm then wants to increase workers’ incentive to put forth effort on task $b$, which it achieves by setting $\theta$ below 1. Which of these effects dominates depends on several parameters, such as the firm’s discount factor, $\delta_F$.

4.4 Performance pay based on an additional performance measure

In the analysis from Section 3 the firm achieves a desirable assignment of workers to jobs when it sets $\theta = 1$. In that case, however, workers in the low-level job focus entirely on the task that is relatively more important in the high-level job. As noted in the introduction, the multitasking literature has identified a similar problem of effort misallocation. In particular, in a situation with imperfect (or distorted) measures of performance, workers focus on the tasks that are easier to measure and that are thus rewarded in an incentive contract based on the imperfect performance measure.

Although the results of our paper and those of the multitasking literature are related, there is one important difference. A promotion rule with $\theta = 1$ induces workers to put forth high effort on the task that is relatively more important in the high-level job. An incentive contract based on an imperfect performance measure induces workers to focus on those tasks that are easy to measure. In many practical situations, the tasks that are relevant in high-level jobs are those that are not easy to formally measure and verify. This means that the promotion tournament with $\theta = 1$ induces workers to focus on those tasks that are typically neglected if performance pay based on an imperfect performance measure were the only instrument to motivate workers. Put differently, if a pay-for-performance scheme based on an imperfect performance measure was complemented by a promotion tournament with $\theta = 1$, the firm could achieve an optimal allocation of efforts across tasks and at the same time a desirable assignment of workers to jobs.

To be more specific, suppose that task $a$ is a "managerial task", whereas task $b$ represents "actual production". Assume that the total number of products that worker $i$ produces in period $t$ is verifiable and denote this performance measure by $p_{it}^v$. It is reasonable to assume that worker $i$ produces more products if he puts forth more effort on task $b$ and if his task-specific ability is higher. Given
that luck may also play a role, we assume \( p^t_i \) is given by
\[
p^t_i = \beta_b (a_{ib} + e^t_{ib}) + \varepsilon^t,
\]
where \( \beta_b \) is a strictly positive constant and \( \varepsilon^t \) is a random variable. Without loss of generality, we assume that the firm pays worker \( i \) a piece-rate \( \gamma_i \) for every unit produced.

In this case it is easy to see that the firm can induce efficient (first-period) efforts and at the same time a desirable assignment of workers to jobs. As shown in Section 4.1, the firm could set \( \theta = 1 \) to solve the problem of assignment, choose \( w_{2H} - w_{2L} \) to induce efficient effort on task \( a \) and use \( w_1 \) to extract all rents from the workers. In addition, it could now use the piece-rate \( \gamma_i \) to induce efficient effort on task \( b \) and, thus, to solve the problem of misallocation of effort across tasks.

The argument in this subsection is related to that of Baker, Jensen, and Murphy (1988). The so-called “Baker-Jensen-Murphy puzzle” argues that it is puzzling that firms use promotions to achieve both incentives and assignment. It would seem more logical for firms to use promotions strictly for efficient job assignment, relying on output-based pay (such as piece-rates or bonuses) to achieve incentives. This would avoid the tradeoff between incentives and assignment that frequently occurs. There is one important difference between that argument and ours. In our model, the firm benefits from using a promotion tournament (with \( \theta = 1 \)) by achieving an efficient assignment of workers to jobs (as in Baker, Jensen and Murphy), but also by motivating workers to put forth effort on task \( a \) (which is different from Baker, Jensen and Murphy). If the performance measure were given by \( p^t_i = \beta_b (a_{ib} + e^t_{ib}) + \varepsilon^t \) and if the firm were to rely solely on \( p^t_i \) to incentivize the workers it could never induce them to put forth effort on task \( a \). The advantage of the tournament is that it rewards effort on the task that is relatively more important in the high-level job (i.e. the managerial task). This means that the tournament is needed to achieve an optimal mix of efforts, and not only for assignment purposes.

5 Testable Implications

The main observable implication of the model is balance in the promotion rule, meaning the firm bases promotions in part on performance in tasks that are predominantly associated with the lower-level job, even though those tasks are de-emphasized in the higher-level job. The likelihood of balance in the promotion rule varies by how easy it is for workers to strategically shirk and how costly
such shirking is to the firm, as well as by how difficult it is for the firm to com-
mit ex ante to the promotion rule. In production settings where the firm has
trouble credibly committing or where strategic shirking is prohibitively costly
to workers and therefore less likely to pose problems for the firm, balance in
the promotion rule should be less likely, meaning the firm more often promotes
workers on the basis of their proficiencies in the tasks that are most heavily
emphasized in the high-level job. In contrast, the greater the threat of strategic
shirking (or the easier it is for the firm to commit) the more likely it is that
the firm bases promotion decisions on performance in the tasks most heavily
emphasized in the lower-level job, thereby creating a more balanced rule. Iden-
tifying production settings in which strategic shirking is likely to be a strong
or weak threat (or where commitment is difficult or easy) would allow these
predictions to be tested. A related implication of balance in the promotion rule
is that the firm may wish to promote workers who have invested in acquiring a
broad range of skills, even those not heavily emphasized in the higher-level job.

Another implication of the model follows from the first sentence of Proposi-
tion 3 and can be tested if the researcher observes variation in the importance
of effort (as opposed to ability) in determining performance, as captured by $q$
in the model. When the relative importance of effort increases, balance in the
promotion rule is more likely to occur. Furthermore, from the second sentence
of Proposition 3, the promotion rule should be more balanced when the cost of
worker effort is low, when the firm heavily discounts the future, and when the
worker lightly discounts the future. An approach for operationalizing these tests
is to use a probit model in which the dependent variable is a binary measure
of whether the promotion rule is balanced, and the underlying latent index is
modeled as a linear function of $q$, $c$, $\delta_F$ and $\delta_W$, or whatever subset of those
four variables is observed in the data. The latent index is modeled as increasing
(decreasing) in $q$ and $\delta_W$ ($c$ and $\delta_F$), so that the probability of balance in the
promotion rule is increasing (decreasing) in $q$ and $\delta_W$ ($c$ and $\delta_F$), as the theory
predicts.

Another observable implication of our model is that Putt’s Law occurs with
positive probability, meaning in some cases the promoted worker is less com-
petent than the non-promoted worker in the tasks that are emphasized in the
lower-level job and that are used as a basis for promotion decisions. Furthe-
more, Putt’s Law is more likely to occur when the promotion rule becomes less
balanced. A further point related to Putt’s Law is that our theory has implica-
tions for the estimated coefficients of performance measures in regressions of
promotion probabilities or wages. When empirical measures of job performance are available they are usually job specific rather than task specific, and they do not always completely capture all dimensions of performance that are relevant for evaluating a worker’s suitability for promotion. Furthermore, it seems fair to say that the dimensions of performance in lower-level positions that are most difficult to measure tend to be those that are emphasized in higher-level jobs (e.g. "managerial potential"). Suppose task $a$ involves "leadership" and task $b$ involves more concrete tasks like sales, and further suppose that the observed lower-level job performance measures fail to capture task $a$. To the extent that firms weight leadership performance heavily in promotion rules (i.e. assign high values of $\theta$) a regression of wages, wage growth attached to promotion, or promotion probability might exhibit small or even negative coefficients on measured performance, for reasons related to Putt’s Law and the fact that the observed job-specific performance measure captures skills in task $b$ more than in task $a$. However, an insight of our model is that strategic shirking (and in particular the firm’s behavioral response to combat that incentive problem) mitigates this estimation issue. The logic is that the threat of strategic shirking encourages the firm to assign greater weight in the promotion rule to tasks that are emphasized in the lower-level job (and that are more easily measured and likely to be reflected in the overall performance measures typically available in datasets). This tightens the positive relationship between observed performance and promotion probability (and consequently between performance and the wage gains attached to promotion). An implication is that when the estimated regression coefficients on performance are small or even negative, this evidence might suggest settings in which strategic shirking is unlikely to be a major concern for the firm (or commitment is particularly hard) so that promotion decisions can be made largely on the basis of intangible factors related to expected managerial performance that are difficult to quantify in standard performance ratings.

Yet another observable implication of the model concerns the degree to which tasks are similar across job levels. Recall that $\tau_2 - \tau_1$ measures the degree to which tasks are similar across levels, with smaller values of this difference indicating greater similarity. There are countervailing effects of similarity on the likelihood of balance in the promotion rule. A higher value for $\tau_2 - \tau_1$ (regardless of whether it results from higher $\tau_2$, lower $\tau_1$, or both) implies that it is more important for the firm to promote the right employee. That is, selection becomes more important since tasks across hierarchical levels differ more strongly. As a result, the firm is more inclined to promote a worker
solely based on his performance in the tasks emphasized in the higher-level job. However, if a higher value of $\tau_2 - \tau_1$ occurs only because $\tau_1$ decreases, the tasks traditionally emphasized in the lower-level job become even more important for performance in that job. This encourages the firm to inject balance into the promotion rule, which in turn encourages workers to increase their effort on those tasks. As noted earlier, which of the two countervailing effects dominates depends on parameters such as the firm’s discount factor.

These predictions concerning the effect of similarity in tasks across job levels on the degree of balance in the promotion rule can, in principle, be tested. Doing so requires data on the tasks used at each job level so that it is possible to measure the extent to which tasks differ across levels. If such information is available over time for a given firm, as in DeVaro, Ghosh and Zoghi (2012), then it is possible to observe not only whether $\tau_2 - \tau_1$ increased, decreased, or stayed the same from period $t$ to period $t+1$ but the nature of the change (e.g. whether an increase in $\tau_2 - \tau_1$ occurred because $\tau_2$ increased, $\tau_1$ decreased, or both changes occurred simultaneously). Identifying the nature of changes in $\tau_2 - \tau_1$ is important for empirical tests, given the countervailing effects noted in the preceding paragraph.

One production setting that might be suitable for testing the prediction concerning $\tau_2 - \tau_1$ is academia, where there are two clearly identifiable tasks (research and teaching) for which performance data are often available over time for individual professors at all job levels. Consider two job levels called "junior" and "senior" faculty. Let $\tau_1$ and $\tau_2$ denote the productivity weights on research for junior and senior faculty, respectively. From a university’s standpoint, an example of an increase in $\tau_2$ (holding $\tau_1$ constant) might be an increase in the availability of external grant funding. Assuming that grants are most likely to be awarded to researchers with clearly established records who are experts in their fields (a status more likely to apply to seniors than to juniors) and who continue to actively publish, the return to the university (in terms of increased likelihood of grant funding) of the marginal publication would increase for seniors relative to juniors. Alternatively, a decrease in $\tau_1$ holding $\tau_2$ constant might represent a decrease in external funding opportunities that are earmarked for early-career scholars. These two different ways in which $\tau_2 - \tau_1$ might increase have different observable implications for the degree of balance in the university’s promotion rule, as explained earlier.

An observable implication of the extension in Section 4.4 is that firms use promotion tournaments and bonuses (or piece rates) simultaneously. There
is some empirical evidence to suggest that these two forms of incentives do in fact go hand in hand. DeVaro and Waldman (2012) note that in private correspondence Daniel Parent reported that in the PSID data used in Lemieux, MacLeod, and Parent (2009) "bonuses and promotions are positively correlated, even controlling for unmeasured worker effects". DeVaro and Waldman (2012) found the same result in the single-firm personnel data used in Baker, Gibbs, and Holmström (1994a,b).

6 Conclusion

A significant volume of theoretical and empirical literature supports the view that promotions create powerful incentives for workers. Most studies in tournament theory assume workers engage in only a single productive task so that there is only one way to impress the boss, and in this case shirking is not an attractive option for workers. But when jobs are comprised of multiple tasks, weak performance on one task can be compensated by stellar performance on another. When this idea is combined with the notion that employers make future job assignments on the basis of observations of workers' earlier task-specific performances, an incentive problem arises whereby workers strategically shirk on some tasks and over-perform on others, so as to enhance their promotion chances. Anticipating the multitasking incentive problem, the firm is encouraged to choose a more balanced promotion rule that assigns some weight to performance in tasks that are important in lower-level jobs but less important in higher-level jobs. Casual observation and anecdotal evidence support the notion that workers distort their efforts across tasks so as to enhance their promotion chances, and that when they fail to do so they are sometimes labeled "too good to promote" and "indispensable" in their current jobs. The theory gives rise to a rich set of testable implications and offers an explanation for Putt’s Law. Although multitasking has been central to the literature on incentive compensation contracts, it has been neglected in the promotions literature despite having important implications as we have just shown.

We conclude by providing two ideas for future research that build on this analysis. To borrow terminology from Waldman (2012), our analysis has focused on the "classic" wage generating mechanism in which the firm strategically chooses wage levels to induce the optimal worker behavior, as in Lazear and Rosen (1981) and Prendergast (1993). An alternative that has received
increasing attention in the literature is the "market-based" wage generating mechanism in which the wage spread from promotion arises as the result of competing bids from other firms in the labor market, as in Waldman (1984). Our first idea for future research is to analyze that case, and our impression is that the central ideas we develop here should also apply in that alternative setting. The reason is that workers care only about the fact that promotions come with wage increases and not about the reason for those wage increases. So as long as workers can allocate effort across tasks, and the resulting performance signals convey useful information to the firm about how to assign workers in the future, the strategic shirking problem and its implications should be present.

A second idea is to integrate the firm’s job design decision into our framework for strategic shirking in promotion tournaments. In our analysis the tasks in each job were exogenously given. But in practice firms purposefully bundle certain tasks together into jobs that are then organized in a hierarchy. Our analysis reveals that there are important incentive considerations that should influence the way in which firms design jobs. For example, from the standpoint of employer learning, it makes sense to design jobs so that performance on tasks in lower-level jobs is highly informative about expected performance in higher-level jobs. But doing so simultaneously exacerbates the strategic shirking problem. In production settings in which the prospect of strategic shirking is particularly costly (or when the firm faces serious commitment problems), the firm might wish to voluntarily forgo valuable information by designing jobs in such a way that performance in lower-level tasks conveys little useful information about performance in higher-level tasks, thereby mitigating or circumventing strategic shirking. This is just one potential example of the interesting implications we believe might emerge from an integration of our analysis with the firm’s job design problem.

Appendix

Concavity of the workers' objective function:

The second-order partial derivatives of worker $i$'s objective function can be determined as

$$
\frac{\partial^2 EU_i}{\partial (e_{ia})^2} = -c + \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial (e_{ia})^2} \delta_W (w_2H - w_2L),
$$

$$
\frac{\partial^2 EU_i}{\partial e_{ia} \partial e_{ib}} = \frac{\partial^2 EU_i}{\partial e_{ib} \partial e_{ia}} = \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial e_{ia} \partial e_{ib}} \delta_W (w_2H - w_2L),
$$

$$
\frac{\partial^2 EU_i}{\partial (e_{ib})^2} = -c + \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial (e_{ib})^2} \delta_W (w_2H - w_2L).
$$

The Hessian is, thus, given by

$$
\begin{pmatrix}
-c + \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial (e_{ia})^2} \delta_W (w_2H - w_2L) & \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial e_{ia} \partial e_{ib}} \delta_W (w_2H - w_2L) \\
\frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial e_{ia} \partial e_{ib}} \delta_W (w_2H - w_2L) & -c + \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial (e_{ib})^2} \delta_W (w_2H - w_2L)
\end{pmatrix}.
$$

To demonstrate that $EU_i$ is strictly concave, we must show that the Hessian is negative definite. This in turn requires that

$$
-c + \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial (e_{ia})^2} \delta_W (w_2H - w_2L) < 0
$$

and

$$
\begin{pmatrix}
-c + \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial (e_{ia})^2} \delta_W (w_2H - w_2L) \\
\frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial e_{ia} \partial e_{ib}} \delta_W (w_2H - w_2L)
\end{pmatrix} - \left( \frac{\partial^2 \Pr \{S_i \geq S_l\}}{\partial e_{ia} \partial e_{ib}} \delta_W (w_2H - w_2L) \right)^2 > 0.
$$

It is easy to see that both these conditions are always fulfilled if $c$ is sufficiently large.

Symmetry of equilibrium

As shown before, the two workers’ first-period expected payoffs are given by

$$
EU_1 = w_1 + \delta_W w_2L - \frac{c}{2} \left( (e_{ia})^2 + (e_{ib})^2 \right) + \Pr \{S_1 \geq S_2\} \delta_W (w_2H - w_2L)
$$
and

\[ EU_2 = w_1 + \delta_W w_{2L} - \frac{c}{2} \left( (e_{2a}^1)^2 + (e_{2b}^1)^2 \right) + \Pr \{ S_2 \geq S_1 \} \delta_W (w_{2H} - w_{2L}). \]

Define \( \Omega := \Pr \{ S_1 \geq S_2 \} \) and notice that \( \Pr \{ S_2 \geq S_1 \} = 1 - \Omega \). As demonstrated in Section 3, first-period efforts enter \( \Omega \) only through the term \( e_{1a}^1 - e_{2a}^1 + (1-\theta) (e_{1b}^1 - e_{2b}^1) \), hence \( \Omega = \Omega \left( e_{1a}^1 - e_{2a}^1 + (1-\theta) (e_{1b}^1 - e_{2b}^1) \right) \). Taking this into account, the first-order conditions with respect to \( e_{1a}^1 \) and \( e_{2a}^1 \) can be stated as

\[
\frac{\partial EU_1}{\partial e_{1a}^1} = -ce_{1a}^1 + \Omega \left( e_{1a}^1 - e_{2a}^1 + \frac{(1-\theta)}{\theta} (e_{1b}^1 - e_{2b}^1) \right) \delta_W (w_{2H} - w_{2L}) = 0
\]

and

\[
\frac{\partial EU_2}{\partial e_{2a}^1} = -ce_{2a}^1 - \Omega \left( e_{1a}^1 - e_{2a}^1 + \frac{(1-\theta)}{\theta} (e_{1b}^1 - e_{2b}^1) \right) (-1) \delta_W (w_{2H} - w_{2L}) = 0,
\]

or equivalently as

\[
\Omega \left( e_{1a}^1 - e_{2a}^1 + \frac{(1-\theta)}{\theta} (e_{1b}^1 - e_{2b}^1) \right) \delta_W (w_{2H} - w_{2L}) = ce_{1a}^1
\]

and

\[
\Omega \left( e_{1a}^1 - e_{2a}^1 + \frac{(1-\theta)}{\theta} (e_{1b}^1 - e_{2b}^1) \right) \delta_W (w_{2H} - w_{2L}) = ce_{2a}^1.
\]

Since the respective left-hand-sides of both conditions are the same, the right-hand-sides must also be the same, and therefore \( e_{1a}^1 = e_{2a}^1 \). Analogously, it can be shown that \( e_{1b}^1 = e_{2b}^1 \).

**Proofs of lemmas and propositions**

**Proof of Proposition 1.** Obviously, workers put forth zero effort because effort is costly, and there is no future benefit to exerting it given that the game ends after period 2. Expected second-period profit is thus given by

\[
\pi_2 = \Pr \{ S_1 > S_2 \} (E[a_{1b} | S_1 > S_2] + \tau_2 (E[a_{1a} | S_1 > S_2] - E[a_{1b} | S_1 > S_2])) + \Pr \{ S_1 > S_2 \} (E[a_{2b} | S_1 > S_2] + \tau_1 (E[a_{2a} | S_1 > S_2] - E[a_{2b} | S_1 > S_2]))
+ \Pr \{ S_1 < S_2 \} (E[a_{2b} | S_1 < S_2] + \tau_2 (E[a_{2a} | S_1 < S_2] - E[a_{2b} | S_1 < S_2]))
+ \Pr \{ S_1 < S_2 \} (E[a_{1b} | S_1 < S_2] + \tau_1 (E[a_{1a} | S_1 < S_2] - E[a_{1b} | S_1 < S_2]))
- w_{2H} - w_{2L}.
\]
Because all skills are drawn from the same distribution, this profit can be re-stated as

\[
\pi_2 = E[a_{1b} | S_1 > S_2] + \tau_2 (E[a_{1a} | S_1 > S_2] - E[a_{1b} | S_1 > S_2]) \\
+ E[a_{2b} | S_1 > S_2] + \tau_1 (E[a_{2a} | S_1 > S_2] - E[a_{2b} | S_1 > S_2]) - w_2H - w_2L.
\]

Proof of Proposition 2. From the expressions characterizing optimal effort, it is straightforward to see that \(e_{ia} \geq e_{ib} \) iff \( 1 \geq \frac{1-\theta}{\theta} \leftrightarrow \theta \geq 0.5 \).

Proof of Lemma 1. As \( \theta \) approaches 1, we obtain

\[
\lim_{\theta \to 1} \frac{\partial \pi_1}{\partial \theta} = \frac{2q \delta W (w_2H - w_2L)}{c} .
\]

By Fubini's theorem, we can write

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g'(a_{ia}) (a_{ib} - a_{ib}) g(a_{ia}) da_{ia}g(a_{ib}) da_{ib}g(a_{ib}) da_{ib}
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g'(a_{ia}) a_{ib}g(a_{ia}) da_{ia}g(a_{ib}) da_{ib}g(a_{ib}) da_{ib}
\]

\[= 0.
\]

Hence,

\[
\lim_{\theta \to 1} \frac{\partial \pi_1}{\partial \theta} = -2q \frac{\delta W (w_2H - w_2L)}{c} .
\]

\[
(1-\tau_1) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(a_{ia}) g(a_{ia}) da_{ia}g(a_{ib}) da_{ib}g(a_{ib}) da_{ib} < 0.
\]
Proof of Lemma 2. Expected second-period profit, as specified in Proposition 1, can be rewritten as

$$\pi_2 = E[\tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} | S_1 > S_2] - w_{2H} - w_{2L}.$$ 

Note that

$$S_1 > S_2 \Leftrightarrow \theta s_{1a} + (1 - \theta) s_{1b} > \theta s_{2a} + (1 - \theta) s_{2b}$$

$$\Leftrightarrow \theta (a_{1a} + e_{1a}) + (1 - \theta) (a_{1b} + e_{1b}) >$$

$$\theta (a_{2a} + e_{2a}) + (1 - \theta) (a_{2b} + e_{2b}).$$

In a symmetric equilibrium, we observe $e_{1a} = e_{2a}$ and $e_{1b} = e_{2b}$ so that

$$S_1 > S_2 \Leftrightarrow \theta a_{1a} + (1 - \theta) a_{1b} > \theta a_{2a} + (1 - \theta) a_{2b}.$$ 

Notice that

$$E[\tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} | S_1 > S_2]$$

$$= \frac{1}{\Pr(S_1 > S_2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b}) g(a_{1a}) da_{1a} g(a_{1b}) da_{1b} g(a_{2a}) da_{2a} g(a_{2b}) da_{2b}$$

$$= 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (\tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b}) g(a_{1a}) da_{1a} g(a_{1b}) da_{1b} g(a_{2a}) da_{2a} g(a_{2b}) da_{2b}.$$
Using Leibniz’s rule, we obtain

\[
\frac{\partial}{\partial \theta} \mathcal{E}[\tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} | S_1 > S_2]
\]

\[
= 2 \int \int \int (-\frac{1}{\theta^2} (a_{1b} - a_{2b})) \cdot \\
(\tau_2 (a_{2a} - \frac{1 - \theta}{\theta} (a_{1b} - a_{2b})) + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b}) \cdot \\
g (a_{2a} - \frac{1 - \theta}{\theta} (a_{1b} - a_{2b})) g (a_{1b}) da_{1b} g (a_{2a}) da_{2a} g (a_{2b}) da_{2b} \\
= -\frac{2}{\theta^2} \int \int \int (a_{1b} - a_{2b}) ((1 - \tau_2) a_{1b} + (\tau_1 + \tau_2) a_{2a} + (1 - \tau_1 + \tau_2 \frac{1 - \theta}{\theta}) a_{2b}) \cdot \\
g (a_{2a}) g (a_{1b}) da_{1b} g (a_{2a}) da_{2a} g (a_{2b}) da_{2b}.
\]

As \( \theta \) approaches 1, the derivative becomes

\[
\lim_{\theta \to 1} \frac{\partial}{\partial \theta} \mathcal{E}[\tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} | S_1 > S_2]
\]

\[
= -2 \int \int \int (a_{1b} - a_{2b}) ((1 - \tau_2) a_{1b} + (\tau_1 + \tau_2) a_{2a} + (1 - \tau_1) a_{2b}) \cdot \\
g (a_{2a}) g (a_{1b}) da_{1b} g (a_{2a}) da_{2a} g (a_{2b}) da_{2b}.
\]

This expression can be rewritten as

\[
-2 \int \int \int (a_{1b} - a_{2b}) ((1 - \tau_2) a_{1b} + (1 - \tau_1) a_{2b}) \cdot \\
g (a_{2a}) g (a_{1b}) da_{1b} g (a_{2a}) da_{2a} g (a_{2b}) da_{2b} \\
-2 \int \int \int (a_{1b} - a_{2b}) (\tau_1 + \tau_2) a_{2a} \cdot \\
g (a_{2a}) g (a_{1b}) da_{1b} g (a_{2a}) da_{2a} g (a_{2b}) da_{2b}.
\]
By Fubini’s theorem, we can write

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (a_{1b} - a_{2b}) (\tau_1 + \tau_2) a_{2a} \cdot g(a_{2a}) g(a_{1b}) da_{1b} g(a_{2a}) da_{2a} g(a_{2b}) da_{2b} 
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a_{1b} (\tau_1 + \tau_2) a_{2a} g^2(a_{2a}) g(a_{1b}) da_{1b} da_{2a} g(a_{2b}) da_{2b} 
\]

\[
- \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} a_{2b} (\tau_1 + \tau_2) a_{2a} g^2(a_{2a}) g(a_{2b}) da_{2b} da_{2a} g(a_{1b}) da_{1b} = 0.
\]

Hence, we have

\[
\lim_{\theta \to -1} \frac{\partial}{\partial \theta} \left[ \tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} \mid S_1 > S_2 \right] 
\]

\[
= -2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (a_{1b} - a_{2b}) \left( (1 - \tau_2) a_{1b} + (1 - \tau_1) a_{2b} \right) g^2(a_{2a}) g(a_{1b}) da_{1b} da_{2a} g(a_{2b}) da_{2b} 
\]

\[
= -2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( (1 - \tau_2) a_{1b}^2 + (\tau_2 - \tau_1) a_{1b} a_{2b} - (1 - \tau_1) a_{2b}^2 \right) g(a_{1b}) da_{1b} g(a_{2b}) da_{2b} g^2(a_{2a}) da_{2a} = 0.
\]

For \( \tau_2 \to \tau_1 \), the expression would collapse to

\[
\lim_{\tau_2 \to \tau_1} \left( \lim_{\theta \to -1} \frac{\partial}{\partial \theta} \left[ \tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} \mid S_1 > S_2 \right] \right) 
\]

\[
= -2 (1 - \tau_1) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( a_{1b}^2 - a_{2b}^2 \right) g(a_{1b}) da_{1b} g(a_{2b}) da_{2b} g^2(a_{2a}) da_{2a} = 0.
\]

It remains to show that the partial derivative of \( \lim_{\theta \to -1} \frac{\partial}{\partial \theta} \left[ \tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} \mid S_1 > S_2 \right] \) with respect to \( \tau_2 \) is strictly positive. We obtain

\[
\frac{\partial}{\partial \tau_2} \left( \lim_{\theta \to -1} \frac{\partial}{\partial \theta} \left[ \tau_2 a_{1a} + (1 - \tau_2) a_{1b} + \tau_1 a_{2a} + (1 - \tau_1) a_{2b} \mid S_1 > S_2 \right] \right) 
\]

\[
= -2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( -a_{1b}^2 + a_{1b} a_{2b} \right) g(a_{1b}) da_{1b} g(a_{2b}) da_{2b} g^2(a_{2a}) da_{2a} 
\]

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Since the $a_{ij}$ are i.i.d., we have $E[a_{1b}a_{2b}] = E[a_{1b}]E[a_{2b}] = E^2[a_{1b}]$. Then, we can restate the last expression as

$$2 \int_{-\infty}^{\infty} \left( E[a_{1b}^2] - E[a_{1b}a_{2b}] \right) g^2(a_{2a}) \, da_{2a},$$

which is obviously positive. ■

**Proof of Proposition 3.** The firm’s expected discounted total profit amounts to $\pi := \pi_1 + \delta_F \pi_2$. We must show that $\lim_{\theta \to -1} \left( \frac{\partial \pi}{\partial \theta} \right) < 0$ under the conditions from the proposition. We know that

$$\lim_{\theta \to -1} \left( \frac{\partial \pi}{\partial \theta} \right) = \lim_{\theta \to -1} \left( \frac{\partial \pi_1}{\partial \theta} \right) + \delta_F \lim_{\theta \to -1} \left( \frac{\partial \pi_2}{\partial \theta} \right).$$

From the previous lemmas we know that $\lim_{\theta \to -1} \left( \frac{\partial \pi_1}{\partial \theta} \right) < 0$, whereas $\delta_F \lim_{\theta \to -1} \left( \frac{\partial \pi_2}{\partial \theta} \right) > 0$. Moreover, from the proof of Lemma 1 it is straightforward to see that $|\lim_{\theta \to -1} \left( \frac{\partial \pi_1}{\partial \theta} \right)|$ is strictly increasing in $q$. Hence, there must be a threshold value $\tilde{q}$ such that $|\lim_{\theta \to -1} \left( \frac{\partial \pi_1}{\partial \theta} \right)| > \delta_F \lim_{\theta \to -1} \left( \frac{\partial \pi_2}{\partial \theta} \right)$ iff $q \geq \tilde{q}$. Since the right-hand-side of the inequality is increasing in $\delta_F$, $\tilde{q}$ must be increasing in $\delta_F$. Moreover, it is straightforward to show that $|\lim_{\theta \to -1} \left( \frac{\partial \pi_1}{\partial \theta} \right)|$ is decreasing in $c$ and increasing in $\delta_W$, whereas $\delta_F \lim_{\theta \to -1} \left( \frac{\partial \pi_2}{\partial \theta} \right)$ is independent of these parameters. Hence, $\tilde{q}$ must be increasing in $c$ and decreasing in $\delta_W$. ■
References


Zábojník, J. (2012), Promotion tournaments in market equilibrium, forthcoming in *Economic Theory*.